The switching effect of environmental taxation within Bertrand differentiated duopoly

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\textbf{A B S T R A C T}

We investigate second-best optimal taxation of the polluting variety of a product in a Bertrand duopoly with differentiated varieties. The analysis provides novel insight on a useful social function of environmental regulation. Besides internalizing the environmental externality, the taxation of the polluting variety improves the matching of consumers and product varieties, and so creates a socially desirable business switching between the differentiated varieties.

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\section{Introduction}

The economic literature has long recognized that the Pigovian task of internalizing marginal social damage is not the only task to be performed by environmental taxes. In most general equilibrium models where pollution is generated under perfectly competitive conditions, the environmental tax is also shown to serve the Ramsey purpose of raising public revenues in order to finance public goods other than the environment (see [37]). In this case, the second-best optimal tax must lie below the marginal environmental damage to achieve an optimal trade-off between the Pigovian and the Ramsey uses of the tax (see [9]). Furthermore, when pollution is produced by an imperfectly competitive industry, the environmental tax proves successful in correcting the distortions associated with market power in the absence of other regulatory devices. It turns out that, in general, the second-best optimal policy under imperfect competition is again to impose a tax lower than the marginal environmental damage. A general intuition provided by Buchanan [10] is that scaling down the tax below the Pigovian level solves the tendency of imperfectly competitive firms to underproduce. This rule holds true not only when pollution is generated by a monopoly (see [22,6]), but also by a symmetric Cournot duopoly [see 23] or an asymmetric Bertrand duopoly with homogeneous product (see [34]). One exception to the rule is that pointed out by Simpson [38]: in the Cournot duopoly with asymmetric costs of production the second-best optimal tax may be greater than marginal damage so as to redistribute output from a less efficient producer to his more efficient rival.

Taking a step forward in this literature, the present analysis pays careful attention to another social function served by taxation of a polluting product. Taxation improves the matching of consumers with heterogeneous tastes and the differentiated varieties of a product by creating a desirable switching effect between these varieties. While the social
benefit deriving from this function is acknowledged in a perfectly competitive context with clean and dirty goods (see [9]), all the aforementioned studies of imperfectly competitive markets fail to account for the substitution possibilities between differentiated varieties that environmental taxation can in fact exploit. The simple reason is that their common focus on homogeneous products cannot capture evidence that consumers perceive differences among the products of firms with different names or logos, thereby treating products as imperfect substitutes. Moreover, the Cournot assumption that prices are chosen by a fictitious auctioneer has long been criticized on the grounds that no such auctioneer exists. In most real-world industries under imperfect competition, prices are chosen by producers who then commit to provide any quantity demanded at those prices. Being more relevant than the Cournot approach in this respect, the Bertrand differentiated products model has successfully been applied to various industries (e.g., [8, 17]).

The literature on environmental taxation of imperfectly competitive markets typically ignores other constraints, such as the budget constraints, although this is a central issue in general equilibrium models (see [9, 26], or [16]). We propose here to investigate the role of second-best optimal taxation in the Bertrand context of two producers setting prices and supplying differentiated varieties of the same product at those prices, when the regulator has a revenue-raising requirement. One variety is called “conventional”, in that it pollutes the environment, while the other variety, which we call “green”, bundles the product with an environmental service to the buyer. By environmental service, we mean the use of recyclable materials, energy efficiency in the whole production process, the adoption of renewable sources of energy, the restricted use of chemicals or the removal of heavy metal, indeed any service that might be certified by a reliable ecolabel. This is consistent with the observation that ecolabeling is increasingly used by firms as a strategic means of vertical differentiation. Besides this vertical differentiation, the green variety also differs from the conventional variety along a horizontal characteristic. Moreover, the introduction of the revenue requirement implies that, rather being simply imposed with a corrective intention, the tax must also be adjusted to account for the cost of raising funds, as commonly experienced by European policymakers in their recent attempts to introduce environmental taxes.

In our Bertrand duopoly with differentiated products, it is no longer obvious that the second-best optimal tax on the conventional variety should be set below the marginal damage. First, scaling down this corrective tax may run against the regulator’s objective of raising more revenue when public funds become scarcer. In our setting, such growing scarcity requires, on the contrary, that the regulator increases the taxes. Moreover, low taxes on the polluting variety can reduce the number of consumers that buy the green variety below the social preferred level. When, for instance, the cost of tying the product with the environmental service is low or the buyers’ aversion to pollution is high, or the environmental damage is severe, it can be socially efficient to encourage buyers to switch to the green variety. In that case, we show that the optimal tax should exceed marginal damage. The intuition for this result is that heavy taxation of the conventional variety makes the polluting producer less aggressive in his pricing strategy. Such a competition-reducing effect implies transfers between Bertrand rivals that achieve a socially desirable split of the market. This result is closely related to that illustrated with a numerical example by Lange and Requate [21] who investigate uniform taxation in a price-setting duopoly with differentiated products. According to these authors, the reason why the second-best optimal tax should be higher than marginal damage is to offset the extreme advantage of the polluter with respect to the private cost. This intuition however, does not always hold in our analysis. When the polluter has much lower production costs than his green rival, the marginal savings from producing the conventional variety are high, which makes this variety fairly valuable from a social standpoint. This provides the regulator with an incentive to reduce the tax on this variety – perhaps even subsidizing it – in order to divert buyers away from the green variety. Moreover, subsidies or low taxes have a competition-enhancing effect by making the polluter as well as his rival more aggressive in their pricing strategy. Thus, scaling down the tax on the conventional variety can also help the regulator to correct the distortion due to Bertrand behavior within the differentiated duopoly. In contrast, raising this tax makes the polluting producer soft in his pricing strategy, thereby decreasing the intensity of price rivalry between producers due to the strategic complementarity of prices. Hence, the “induced” effects of taxation recognized by Myles [28, 29] for imperfectly competitive markets still occur when the tax must also correct for the environmental externality.

Besides the regulator’s task of internalizing the two negative externalities generated by pollution and Bertrand pricing, attention is also paid to how second-best optimal taxation of the conventional variety is altered by the regulator’s revenue-raising requirement. When circumstances require soft (severe) taxation, the optimal tax rises (falls) in response to an increase in the regulator’s need for revenue. The intuition departs from that underlying the result of a fall in the optimal Pigovian tax in response to increasing revenue needs, established by general equilibrium models under perfect competition (see, for instance, [26]). When heavy taxation of the polluting producer is required under Bertrand pricing with differentiated products, scaling down the tax makes this producer more aggressive in his pricing strategy, thereby increasing the profit earned from the conventional variety as well as the surplus extracted by the regulator.

The present analysis has particular relevance at a time when policymakers need a sound understanding of how the design of environmental taxation affects economic outcomes. Recent examples include the so-called “plastax” imposed on

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1 In the words of Chamberlin [11, pp. 56–57], “virtually all products are differentiated, at least slightly.”
2 In an attempt to reconcile the Cournot model with this evidence, Kreps and Scheinkman [20] analyze a two-stage game in which firms choose capacities and then prices. Under a particular form of quantity rationing, they find that Cournot equilibrium outputs obtain.
3 See [1, 14]. Deltas et al. [14] note that the strategy of differentiating product with a green variety is now commonly used by market leaders such as Unilever, Toyota, Honda or MacDonalds, for the obvious purpose of increasing market power.
plastic-bag consumption by the Irish Government in 2002, the 2010 “showroom tax” on cars with high carbon dioxide emissions in the United Kingdom, the closely related system of “bonus-malus” recently implemented by the French government in the automobile industry, the “gas-guzzler tax” added in 1978 to the cost of new vehicles that do not reach certain fuel economic standards in the United States, and the pesticide and fertilizer taxes in Scandinavian countries. All of these examples highlight at least three intentions of regulators: the corrective intention in Pigou’s spirit, the revenue requirement and the incentive to switch markets toward cleaner alternatives such as leguminous crops to replace artificial fertilizers, fuel-efficient car models and biodegradable or high-quality reusable shopping bags.

2. Regulation of the Bertrand differentiated duopoly

2.1. Model presentation

Let us consider a market in which two Bertrand rivals supply a product that can be defined along two characteristics, namely variety and an environmental service. The range of potential varieties is represented by a Hotelling’s segment of unit length. Only two technically feasible varieties can be produced (one per producer). Not only they are located at the extremes of the segment, but they also differ in the damage caused to the environment. Given this positioning, the two producers simultaneously choose their prices. The assumption of exogenous maximal differentiation on Hotelling’s segment is consistent with the principle stated by d’Aspremont et al. that producers seek to provide products that differ as much as possible according to a horizontal characteristic in order to relax price rivalry.

Variety 1, located at the left extreme of the segment, is the conventional variety of the product. This variety is sold without environmental service, thereby generating polluting externalities. Variety 2, located at the right extreme, is the “green” substitute of the product. This variety is tied to an environmental service that prevents or eliminates all polluting externalities. While buyers do not rank varieties in the same way, everything else being equal all buyers agree that a variety is worse the more polluting it is. Following the distinction made by Philips and Thisse, variety represents here a horizontal attribute of the product, such as the design, color or style, whereas the environmental service is a vertical attribute of the product.

If horizontal differentiation has the traditional meaning of spatial differentiation, then the duopoly might involve two stores located far apart in a city for the sake of reduced price competition. Recalling the “plastax” example of environmental regulation mentioned in Introduction, the vertical dimension added to the spatial context would be that one store provides buyers with cheap plastic bags whereas its rival proposes more expensive biodegradable bags. Moreover, the actual use of pesticide and fertilizer taxes in the agricultural sector suggests further motivation for the idea of combining both horizontal and vertical differentiations into our model. Consider two varieties of agricultural foods supplied respectively by conventional agriculture (variety 1) and sustainable farming (variety 2). Both varieties can hardly attract similar tastes due to different production techniques or soil cultivation, hence there will be no ranking among all consumers for the two varieties (horizontal characteristic). Moreover, conventional agriculture traditionally makes heavy use of pesticides and fertilizers, all of which have serious consequences on water quality, soil erosion, biodiversity and eutrophication (see ). In contrast, sustainable farming requires environmental services such as restricted use of chemicals, cleanup of polluted water or soil remediation. Presumably, most consumers aware of the environmental friendliness of such services prefer to have them bundled with the purchased food (vertical characteristic).

Variety is sold at price by producer who incurs a constant marginal cost of production . The conventional-polluting variety of the product is assumed to be less costly to produce than the green variety, that is, . This is the most common hypothesis in the environmental economics literature, which might reflect the need to use more costly technologies for saving fossil energies or improving fuel efficiency.

Let denote the output of the polluting producer. For analytical convenience, the damage from pollution generated by the conventional variety will be represented by the quadratic function with .

Both varieties provide buyers with the same gross surplus of value . Buyers preferences vary along two dimensions. First, the location of a buyer represents his taste for the product, that is, the buyer’s ideal variety. When purchasing one variety, buyers pay the same matching cost per unit of distance, which represents the utility loss for not purchasing the ideal product. Second, all buyers are assumed to prefer the product to be more environmentally friendly. For expository simplicity, we assume that all consumers will agree to pay exactly the same additional amount

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4 Italy also levies a tax on plastic bags.

5 In the Hotelling tradition, this limited spectrum of varieties on offer is implicitly related to the presence of fixed costs (capital, personnel, research and development, etc.).

6 The assumption of firms equidistant locations on Salop’s circle applies the same principle in the oligopoly model of horizontal product differentiation with free entry.

7 This assumption follows the recent approach in environmental literature that considers environmental friendliness as a quality attribute of a product. Modelling variants of this idea can be found in Arora and Gangopadhyay, Bansal and Gangopadhyay, Cremer and Thisse, Mason, Moraga-Gonzalez and Padron-Fumero and Rothfels.

8 This is particularly true for conventional versus organic food such as fruits, vegetables or wine, the taste of which highly depends on how the soil is cultivated. Another example of horizontal differentiation between conventional and organic food is beef. Because it is lower in fat, organic beef fed with grass does not taste like conventional beef dosed with growth-boosting hormones and fed with grain.
for the environmental service tied to the green variety. This will be captured by an index of environmental concern \( \beta > 0 \) measuring the buyers’ utility loss when purchasing the conventional variety, due to a common aversion to pollution.

 Buyers are assumed to purchase at most one unit of variety. It is also assumed that \( v \) is large enough both to guarantee that the full coverage of the market is socially efficient (see Assumption 2 below) and for all buyers to find a variety for which their surplus is positive in equilibrium (see Assumption 4 below). A buyer located at \( x \in [0,1] \) derives a surplus \( v - \beta - p_1 - tx \) from purchasing variety 1 at price \( p_1 \), and \( v - p_2 - t(1-x) \) from purchasing variety 2 at price \( p_2 \). The polluting producer’s market share \( X \) corresponds to the marginal buyer who is indifferent between both varieties. Thus, \( X \) solves equation:

\[
v - \beta - p_1 - tX = v - p_2 - t(1-X).
\]

It follows that the demand functions for the conventional variety and the green variety are respectively given by:

\[
X_1(p) = \frac{1}{2} + \frac{p_2 - p_1 - \beta}{2t},
\]

\[
X_2(p) = 1 - X_1(p) = \frac{1}{2} + \frac{p_1 - p_2 + \beta}{2t},
\]

where \( p \equiv (p_1, p_2) \) is the vector of prices.

 Before studying producers’ choice of price in the differentiated duopoly left to itself, we briefly derive what should be the socially optimal allocation of varieties among buyers. Social welfare is represented by the sum of buyers’ surplus and firms’ profits less the sum of the social costs entailed by the environmental damage and the imperfect matching of buyers and varieties that occur due to design of the product. The welfare function is

\[
W(X) = v - \beta X - c_1 X - c_2 (1-X) - D(X) - T(X),
\]

where \( T(X) \) denotes total matching cost which can be explicitly expressed here as \( T(X) = \int_0^X tx \, dx + \int_X^1 t(1-x) \, dx = t(x^2 + (1-x)^2)/2 \). As \( T(X) = 2t(X-1/2) \), the function \( T(X) \) is minimized at the market center, meaning that the matching cost of one more buyer choosing the conventional (green) variety is positive for all \( X \) such that \( 0 < X < 1/2 \).

 In what follows, it will be useful to work with \( s \equiv c_2 - c_1 - \beta \) that reflects the marginal savings in production costs of one buyer purchasing the conventional variety, net of his aversion to pollution (from now on, \( s \) will be abbreviated as NMS—for net marginal savings—from producing the conventional variety). Clearly, from (4), the efficient allocation of varieties requires that \( s = T(X^*) + D(X^*) \), i.e., the NMS from producing the conventional variety must exactly offset the sum of marginal social costs. From the social standpoint, the market should be split at the location \( X^* \) such that

\[
X^* = \min \left\{ 1, \frac{t + s}{2t + \delta} \right\}.
\]

Note that supplying both varieties on the market is socially efficient provided that \( 0 < X^* < 1 \). In what follows, we will assume that this condition is satisfied, thereby restricting attention to parameter values such that the following twofold inequality holds.

**Assumption 1.** \( -t < s < t + \delta \).

As previously mentioned, we also assume that \( v \) is large enough to guarantee that the full coverage of the market is socially efficient.

**Assumption 2.** \( v > \max\{\beta + c_1 + (t + s)/(2t + \delta), c_2 + t(1 + \delta - s)/(2t + \delta)\} \).

### 2.2. Bertrand equilibrium outcome for the unregulated market

We now turn to the Bertrand equilibrium outcome for the unregulated market with differentiated varieties. The profit earned from variety \( i \) is \( (p_i - c_i)X_i(p) \) and the first-order conditions are given by:

\[
(p_i - c_i)\frac{\partial X_i(p)}{\partial p_i} + X_i(p) = 0, \quad i = 1, 2.
\]

One can easily check that second-order conditions are satisfied. Let \( \eta_i(p) = -(\partial X_i(p)/\partial p_i) / (p_i / X_i) \), \( i = 1, 2 \), denote the price elasticities of demand for the two varieties. First-order conditions can be rewritten in the usual way to show that the

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9 This assumption simplifies considerably the study of global second-order conditions and both existence and uniqueness of price equilibrium in a context where two horizontally differentiated varieties also differ along a vertical characteristic. As suggested by one referee, we could also incorporate heterogeneity among buyers in the degree of environmental concern by considering that each buyer has a pollution valuation \( \delta \beta \), with \( \delta \in [0,1] \). Under our assumptions, such an extension of the model would make “horizontal dominance prevail” in the sense of Neven and Thisse [30]. These authors however, pointed out that establishing existence as well as uniqueness of price equilibrium may be problematic in the case of horizontal dominance. Greker [18] is a useful reference for a model that combines horizontal differentiation in product taste and vertical differentiation in environmental quality. A somewhat different view is proposed by Eriksson [15] who investigates a model in which environmental quality is partly a horizontal characteristic of the product. See also Conrad [12] for an alternative way of incorporating environmental concern into individual preferences.

10 Strict convexity of \( D(X) \) ensures that second-order conditions are satisfied.
Lerner index is equal to the inverse of the price elasticity of demand, which implies that market power is a decreasing function of the price elasticity of demand:

\[
\frac{p_i - c_i}{p_i} = \frac{1}{\alpha_i(p)}, \quad i = 1, 2. \tag{7}
\]

These conditions yield two reaction functions, which can be solved for the pair \( p^g = (p_1^g, p_2^g) \) of Bertrand–Nash equilibrium prices.

**Proposition 1.** The Bertrand equilibrium outcome for the unregulated market is characterized by the following equilibrium profit margins:

\[
p_1^g - c_1 = \frac{s+3t}{3}, \tag{8}
\]
\[
p_2^g - c_2 = \frac{3t-s}{3}. \tag{9}
\]

At these prices, the demanded quantities for the conventional and the green varieties are respectively:

\[
X_1(p^g) = \min \left\{ 1, \frac{1}{2} + \frac{s}{6t} \right\}, \tag{10}
\]
\[
X_2(p^g) = \max \left\{ 0, \frac{1}{2} - \frac{s}{6t} \right\}. \tag{11}
\]

If \( s < 3t \), then the corresponding equilibrium profits for the conventional and the green varieties are respectively given by:

\[
\Pi_1^g = \frac{(s+3t)^2}{18t}. \tag{12}
\]
\[
\Pi_2^g = \frac{(3t-s)^2}{18t}. \tag{13}
\]

The expressions above clearly show that \( s \) and \( t \) have contrasting effects on the Bertrand equilibrium outcome. An increase in NMS from producing the conventional variety both raises the profit of the conventional variety and lowers that of the green variety, while an increase in matching cost raises prices and profits for both varieties. A rise either in aversion to pollution \( \beta \) or in the cost \( c_1 \) of producing the conventional variety, as well as a decrease in the cost of providing the tied-in environmental service, reduces \( s \), thereby making the neighboring clientele, at the same time, less captive of the polluting producer and more captive of the green producer. This allows the latter to charge a higher price for his variety and obtain a higher profit margin. The result that producers obtain higher profits when \( t \) rises is the usual one that the degree of market power is greater for both producers with a higher degree of horizontal product differentiation, because both demands become less elastic.

Note that the case \( \Pi_2^g > \Pi_1^g \), which only occurs when \( s < 0 \), corresponds to the situation where the green variety supplied by the market left to itself is both beneficial for the environment and more profitable than the conventional variety. Such a “win-win” situation in the absence of regulation is closely related to the variant of the Porter hypothesis investigated by André et al. [2].

To bypass exit problems, we will make the following assumption from now on.

**Assumption 3.** \( s < 3t \).

Hence, we have \( 0 < X_1(p^g) < 1 \) so that each producer gets some part of the market in Bertrand equilibrium.

As previously mentioned, we also assume that \( \nu \) is large enough for all buyers to find a variety for which their surplus is positive in Bertrand equilibrium

**Assumption 4.** \( \nu > \max \{ \beta + c_1 + t(3t+s)/6t, \ c_2 + t(3t-s)/6t \} \).

Within Bertrand differentiated duopoly, the profit-maximizing behavior induces producers to exploit their power over price and maintain a profit margin above marginal cost. Moreover, imperfect competition provides producers with a private incentive to steal the rival’s clientele, thereby restricting his output more than is socially desirable. In general, the resulting allocation of varieties among buyers at Bertrand equilibrium is biased away from the social optimum. This inefficient share of the market is generated by a “business-stealing” effect similar to that emphasized by Mankiw and Whinston [25] for imperfectly competitive markets, regardless of whether products are homogeneous or differentiated.\(^{11}\)

The key point here is that Bertrand rivalry with differentiated varieties implies transfers between producers that do not

\(^{11}\) The business-stealing effect is generally related to entry issues. When a new firm enters the market, it brings new output that increases the welfare. However, existing firms decrease their output in response to the entry. Therefore, the final impact on welfare depends on whether the additional output is higher than the one “stolen” from existing firms.
correspond to a benefit to society. One producer is always “stealing” the other’s clientele in Bertrand equilibrium, and the resulting market split never coincides with the socially optimal one except by chance, if \((3t+s)\delta-4st = 0\) so that \(X_t(p^t) = X^t\). In particular, the specific condition under which the Porter hypothesis holds here, that is \(s < 0\), does not necessarily imply that production of the green variety is socially optimal.

To state the next corollary, we define \(\delta = 4st/(3t+s)\) as the threshold value of \(\delta\) for which the market split in Bertrand equilibrium is socially optimal. Easy calculations show that, under Assumption 2, we have \(s-t < \delta\).

**Corollary 1.** In the absence of regulation:

- If \(\delta > \delta\), then the polluting (green) producer’s market share is larger (lower) than that implied by the socially optimal solution;
- If \(s-t < \delta < \delta\), then the green (polluting) producer’s market share is larger (lower) than that implied by the socially optimal solution.

### 2.3. Optimal tax policy

Let us now investigate the second-best optimal tax that a benevolent regulator should impose on the polluting variety to allocate the two varieties among buyers. The regulator’s problem is to choose a tax \(t\) on each unit of the conventional variety, that maximizes welfare given behavior à la Bertrand within the differentiated duopoly. The variable \(t\) will take negative values if it turns out to be a subsidy. Let \(p^t(t) = (p^t_1(t), p^t_2(t))\) denote the pair of equilibrium prices resulting from Bertrand rivalry between the taxed conventional producer and the untaxed green one. Then, the profit earned from the conventional variety is \(\Pi_t(p^t(t)) = (p^t_1(t) - c_1 - t)X_t(p^t_1(t))\) and \(p^t_1(t)\) must solve the first-order condition for profit maximization of the polluting producer:

\[
(p^t_1(t) - c_1 - t)\frac{\partial X_t(p^t_1(t))}{\partial p^t_1(t)} + X_t(p^t_1(t)) = 0.
\]

Note that substituting \(c_1 + t\) into the expressions of \(p^t_1\) and \(p^t_2\) given by (8) and (9) yields the following equilibrium prices:

\[
p^t_1(t) = c_2 + 2c_1 - \beta + 3t + 2t,
\]

\[
p^t_2(t) = c_1 + 2c_2 + \beta + 3t + t,
\]

and the corresponding equilibrium profits:

\[
\Pi_t(p^t_1(t)) = \frac{(s+3t-t)^2}{18t},
\]

\[
\Pi_t(p^t_2(t)) = \frac{(3t-s+t)^2}{18t}.
\]

From the expressions above, a tax softens the intensity of price rivalry within the duopoly, while a subsidy increases this rivalry. As taxes raise the costs incurred by the polluting producer, they lower his profit and increase the profit of the green producer. In this regard, taxes make the polluting producer soft, whereas subsidies make him tough. Thus, the choice between a tax and a subsidy depends here on whether or not the regulator is willing to strengthen the green producer. Moreover, while setting the tax, the regulator ought to be careful not to drive either variety out of the market since the availability of both varieties is socially efficient by Assumption 1. This participation constraint requires

\[
0 < X_t(p^t_1(t)) < 1,
\]

where \(X_t(p^t_1(t)) = (s+3t-t)/3\).

The untaxed green producer equates marginal revenue and marginal cost in accordance with (6). Moreover, using (7), Eq. (14) can be rewritten to express the Lerner index:

\[
\frac{p^t_1(t) - c_1}{p^t_1(t)} = \frac{\tau}{p^t_1(t)} + \frac{1}{\partial_t(p^t_1(t))}.
\]

The regulator employs the tax on the polluting variety not only to internalizeize the environmental externality but also to raise revenue which might be used, for instance, to decrease other distortionary taxes. The shadow price of public funds will be represented by \(\lambda > 0\), i.e., any dollar of revenue reduces the cost of regulation by one dollar and thus reduces the deadweight loss of distortionary taxation by \(1+\lambda\) dollars.\(^{12}\) Hence, social welfare is

\[
W(t) = v + \beta X_t(p^t_1(t)) - (c_1 + t)X_t(p^t_1(t)) - c_2X_2(p^t_1(t)) - D(X_t(p^t_1(t))) - T(X_t(p^t_1(t))) + (1 + \lambda) t X_t(p^t_1(t)).
\]

\(^{12}\) Moraga-Gonzalez and Padron-Fumero [27] mention the shadow price of public funds in a duopoly model of environmentally differentiated product, though they assume it to be zero.
A detailed analysis of welfare maximization is relegated to Appendix 1. As a result, the second-best optimal tax $\tau^*$ must satisfy the following first-order condition:

$$p_B^B(\tau^*) = \frac{p_B^B(\tau^*) + D(X_1(p_B^B(\tau^*))) - \lambda \tau^* + \lambda}{\frac{\partial p_B^B(\tau^*)}{\partial \tau} - \frac{\partial p_B^B(\tau^*)}{\partial \tau}}.$$  \hfill (22)

As the profit margin expressed in (22) should equal that given by (20), we obtain the following formula which strongly resembles those derived by Sandmo [37] or Bovenberg and de Mooij [9] for a competitive industry:

$$\tau^* = \frac{\lambda}{(1 + \lambda) \lambda^1(p_B^B(\tau^*))} \left( \frac{\partial p_B^B(\tau^*)}{\partial \tau} - \frac{\partial p_B^B(\tau^*)}{\partial \tau} \right) + \frac{1}{(1 + \lambda)} \left[ D(X_1(p_B^B(\tau^*))) + \frac{p_B^B(\tau^*)}{\lambda^1(p_B^B(\tau^*))} - \frac{p_B^B(\tau^*)}{\lambda^1(p_B^B(\tau^*))} \right].$$  \hfill (23)

In Sandmo [37], the optimal tax is a weighted average of the marginal damage and a Ramsey term which reflects the regulator’s revenue requirement. The formula given by (23) departs from Sandmo [37] in that the regulator employs here the tax not only to satisfy his revenue requirement (the first term in the right-hand side of (23)), but also to simultaneously correct for two negative externalities captured by the second term between brackets in the right-hand side of (23): first, the pollution externality, and second, the externality exerted on the society by the Bertrand behavior of the differentiated duopoly. Part of the optimal tax $\tau^*$ achieves an optimal trade-off between both of these distortions.

When the budget requirement is not asking for much, shadow costs of public funds become small ($\lambda \rightarrow 0$), the optimal tax reduces to the second term that reflects the conflict between the welfare gain from internalizing the marginal damage from pollution and the inefficiency due to Bertrand pricing. This differs from the monopoly inefficiency investigated by Lee [22] and Barnett [6] in that the regulator can here mobilize the force of Bertrand pricing to generate a socially beneficial business switching. The regulator must take into consideration that the green producer will react to any price increase (decrease) of the taxed variety by raising (lowering) the price of the green variety, owing to the strategic complementarity of prices which characterizes Bertrand pricing with differentiated varieties. Hence, regulation can make the polluting producer more or less tough in the pricing game. The term $p_B^B(\tau^*)/\lambda^1(p_B^B(\tau^*))$ in the right-hand side of (23) captures the effect on social welfare of the duopoly pricing behavior. It is akin to the imperfect competition “correction” term shown by Myles [29] to modify the Ramsey tax rule in the absence of environmental externality for markets with imperfect competition. Whenever this term is negative, it corresponds to a welfare loss that must be offset by lowering the tax. As in Sandmo [37], it is conceivable that the tax cut becomes so sharp that the optimal policy imposes to subsidize the conventional variety.

An increase in $\lambda$ means that public revenues become scarcer and the weight of the Ramsey term $\lambda/(1 + \lambda)$ rises. When $\lambda$ tends to infinity, the regulator focuses more on generating revenues and less on internalizing all the externalities present in the industry. The tax is then used less for externalities, including the environmental one, and more to raise revenue in the most efficient way: $\tau^*$ approaches $(p_B^B(\tau^*)/\lambda^1(p_B^B(\tau^*)))/(\partial p_B^B(\tau^*)/\partial \tau - \partial p_B^B(\tau^*)/\partial \tau)$, that is, from (20), the profit margin of the taxed producer corrected to account for the effect of taxation on the price of the green variety. In such a case, the only aim of the regulator is to extract the whole surplus of the polluting producer.

Proposition 2 provides an explicit solution for the second-best optimal tax, calculations of which can be found in Appendix 1.

**Proposition 2.** Implementation of the optimal allocation of varieties among buyers is achieved by $\tau^*$ such that

$$\tau^* = \frac{(3t + s)(\delta + 6\lambda t) - 4st}{\delta + (1 + 6\lambda)2t},$$  \hfill (24)

provided that $\lambda > (s - t - \delta)/(9t - s)$.

The condition $\lambda > (s - t - \delta)/(9t - s)$ guarantees that the optimal tax given by (24) satisfies the participation constraint (19) which can easily be turned into $s - 3t < \tau^* < 3t + s$.

The range of parameter values in Fig. 1 is restricted to satisfy Assumptions 1 and 3. It illustrates that optimal regulation imposes either a tax or a subsidy on the polluting producer depending on the combination of two exogenous parameters, namely the index of marginal damage ($\delta$) and the NMS ($s$) previously defined as the difference between the extra cost of producing the green variety ($c_2 - c_1$) and the buyer aversion to pollution ($\beta$). The function $\delta(s) \equiv 2t(2s - 3\lambda (3t + s))/(3t + s)$ is defined as the value of $\delta$ for which the optimal tax is zero. Fig. 1 is drawn for $\lambda < 1/6$ and depicts $\delta(s)$ as the borderline between the two regions I and II$_a + II_b$. In region I, the conventional variety is socially worthwhile due to large NMS from producing this variety (either because of a large cost advantage for the polluting producer, an expensive tied-in environmental service or a low aversion to pollution among buyers) as well as low environmental damages, therefore the regulator finds it more beneficial to give subsidies to the conventional producer. This makes him tough in the pricing game and raises the green producer’s incentive to respond aggressively by scaling down the price of the green variety. Such a competition-enhancing effect is socially beneficial in that it mitigates the distortion due to Bertrand rivalry, as well as increasing production of the conventional variety. By contrast, parameter values in region II$_a + II_b$ are such that a positive tax is imposed on the conventional variety to encourage buyers to switch towards the green variety. As previously shown, this also makes the polluting producer less aggressive in his pricing strategy, thus taxation in that case reduces competition between producers in a way that transfers production to the green producer.
In a model of Bertrand duopoly with two differentiated varieties of the same product, namely a conventional variety and a green substitute, we have examined taxation of the conventional variety when the regulator has a revenue-raising requirement. We find that the second-best optimal tax consists of a revenue-raising component and a component that...
helps correct for the two negative externalities due to pollution and noncompetitive pricing. The main role of environmental regulation here is to improve the matching of buyers and products that suit their tastes in a socially beneficial way. The key point is that induced effects of taxation on Bertrand equilibrium pricing creates an efficient business switching between the differentiated varieties.

The model allows for exploring the applicability of second-best optimal taxation in various economic contexts—including the case where the regulator needs revenue. As a result, taxation of the conventional variety ought to be soft when the conventional variety is socially worthwhile. Such a context prevails when either the polluting producer has a large production cost advantage relative to his green rival, or the environmental service bundled with the green variety is costly, or buyers have a low aversion to pollution. In addition to slightly encourage buyers to switch towards the green variety, soft taxation mitigates Bertrand competition. Faced with soft taxation, the polluting producer refrains from being too friendly in the pricing game, which helps the regulator reduce the distortion due to Bertrand rivalry. Moreover, in response to an increasing need of revenue arising in this context, the regulator is shown to raise the optimal tax on the conventional variety.

On the other hand, the green variety may be highly valuable to society for various reasons: R&D succeeds in lowering the cost of green production techniques, the tied-in environmental service becomes cheap or there is a shift toward more environmental concern among buyers due to a greater influence of green lobbies. In this context, boosting the demand for the green variety requires heavy taxation of the conventional substitute. Obviously, this has a competition-reducing effect since it makes both producers soft in the pricing game. The simple role of second-best optimal taxation is then to achieve an optimal trade-off between the distortion due to Bertrand pricing and the correction of the environmental externality. If, furthermore, circumstances require the optimal tax to exceed marginal damage, then a rise in required tax revenue leads to a decrease in the optimal tax on the conventional variety, which helps the regulator extract more surplus from the polluting producer.

Our analysis of second-best environmental taxation departs from previous works on taxation under imperfect competition in two respects. First, we allow for product heterogeneity and taste differences among buyers. This sheds light on the substitution effects of taxation on demand between the conventional-polluting variety of a product and its green substitute whenever it exists. Second, we assume that the environmental regulator has to meet a revenue requirement. This focus shows that taxes may be levied not so much because they correct dirty production through changing relative prices, but because they generate revenues in addition to encourage buyers to switch towards green consumption. Both these motives are apparently important to European policymakers, who experiment with various forms of environmental taxation in the ever-more demanding context of low economic growth and growing public deficits.

While the assumptions of product heterogeneity and price rivalry better conform to real-world observations than those underlying the standard Cournot approach, our model is still quite stylized. Two limits are worth mentioning. First, total output is fixed in the tradition of Hotelling [19]. While this assumption allows us to focus on the effect of environmental taxation on substitution between varieties, it ignores the restrictive effect of taxation on total output usually reported by previous models with homogeneous products. Second, we assume away the possibility of endogenous entry, as well as the related problem of endogenous product positioning. In this regard, our analysis can be interpreted as having a short-run focus. Insofar as tying an environmental service with the product is a strategic device, it could be used by a firm to pack the product space and deter entry. Addressing this issue would require the introduction of considerable complexity into the present model.

Another potential extension of this research would be to consider non-cooperative regulation of pollution and prices in the spirit of Baron [7]. In such a case, an environmental agency and a public utility commission, or an environment...
ministry and a trade ministry, would be responsible for regulating pollution and prices separately. Our model suggests that the objectives of the two regulators might be in conflict since, in some circumstances, the green producer is guilty of stealing the polluting producer's clientele from the social standpoint, although he has environmentally desirable behavior.

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Appendix

Differentiating welfare (21), we get the following first-order condition

\[
(s-D(X_1(p_b^{(s)}))) - T'(X_1(p_b^{(s)})) + \lambda \tau + \frac{\partial X_1(p_b^{(s)})}{\partial \tau} + \lambda X_1(p_b^{(s)}) = 0, \tag{26}
\]

where

\[
\frac{\partial X_1(p_b^{(s)})}{\partial \tau} = \frac{\partial X_1(p_b^{(s)})}{\partial p_b^{(s)}} \cdot \frac{\partial p_b^{(s)}}{\partial \tau} + \frac{\partial X_1(p_b^{(s)})}{\partial p_b^{(s)}} \cdot \frac{\partial p_b^{(s)}}{\partial \tau} = \frac{\partial X_1(p_b^{(s)})}{\partial p_b^{(s)}} \left( \frac{\partial p_b^{(s)}}{\partial \tau} - \frac{\partial p_b^{(s)}}{\partial \tau} \right).
\]

Furthermore, from (2), we can derive the following property which subsequently proves useful.

For all \( p \), we have

\[
T'(X_1(p)) = p_2 - p_1 - \beta. \tag{27}
\]

Thus, from (2), \( T'(X_1(p_b^{(s)})) = 2t(X_1(p_b^{(s)})) - \frac{1}{2} \) can be replaced by \( p_b^{(s)}(\tau^*) - p_b^{(s)}(\tau^*) - \beta \) in (26) to yield

\[
(p_b^{(s)}(\tau^*) - c_1) - (p_b^{(s)}(\tau^*) - c_2) - D(X_1(p_b^{(s)}))) + \lambda \tau = - \lambda X_1(p_b^{(s)}) \left/ \frac{\partial X_1(p_b^{(s)})}{\partial \tau} \right. \tag{28}
\]

Using the first-order for profit maximization given by (7) for the green producer, we can introduce price elasticities of demand and rewrite (28) as (22).

Using (15) and (20), we get

\[
p_b^{(s)}(\tau^*) - c_1 = \tau^* + \frac{p_b^{(s)}(\tau^*)}{\epsilon_1(p_b^{(s)}(\tau^*))} = \frac{s + 3t + 2\tau^*}{3}. \tag{29}
\]

Moreover, combining (16) with (7) yields

\[
p_b^{(s)}(\tau^*) - c_2 = \frac{3t - s + \tau^*}{3}. \tag{30}
\]

To obtain (24), we can now replace the price elasticities of demand in (23) by the expressions provided by (29) and (30), as well as \( D(X_1(p_b^{(s)})) \) for \( \partial X_1(p_b^{(s)}) \) where \( X_1(p_b^{(s)}) = (3t + s - \tau^*)/6t \) from (10) in Proposition 1.

Furthermore, the optimal tax given by (24) ought to satisfy the participation constraint (19) which is equivalent to the twofold inequality \( s - 3t < \tau^* < 3t + s \). Straightforward calculations show the following results:

1. \( \tau^* < 3t + s \Longleftrightarrow \tau^* \in (1 + \lambda) > 0 \), which always holds under Assumption 1 and 3;
2. \( s - 3t < \tau^* \Longleftrightarrow \delta > s - t + \lambda (s - 9t) \), which can be rewritten \( \lambda > (s - t - \delta)/(9t - s) \) since \( s - 9t < 0 \) by Assumption 3.

References

[36] S. Salop, Monopolistic competition with outside goods, Bell J. Econ. 30 (1979) 141–156.